Study of positioning estimation with user position affected by outlier: a case study of moving-horizon estimation filter

Moath Awawdeh[1], Tarig Faisal Ibrahim[1], Anees Bashir[1,2], Flower M. Queen[2]
[1]Department of Electrical Engineering, Faculty of Engineering Science, Higher Colleges of Technology, Abu Dhabi, UAE
[2]Department of Electrical Engineering, Faculty of Engineering, Noorul Islam Centre for Higher Education, India

Article Info

Article history:

Received Sep 02, 2021
Revised Feb 25, 2022
Accepted Mar 05, 2022

Keywords:

Global positioning system
Moving-horizon estimation
Outliers

ABSTRACT

Many applications which require accurate location point positioning systems utilize global position system for pseudosciences. One of the main challenges faced by the system occurs due to the inherent errors that are a resultant of outliers. This considerably reduces the accuracy of the observations of global position system device. In this paper, we briefly introduce the problem of position estimation when the pseudo range measurements have an outlier. Moving horizon estimation algorithm has been adapted for the simulation result compared with the extended Kalman filter model, which is still imperfect for the case of outlier. The point at which a pseudo range becomes an outlier is considered at a fixed time instance. A simulation example is presented using an existing model with a moving horizon estimator and an extended Kalman filter. The moving horizon filter turns to be more robust than Kalman filtering with presence of outlier under certain choice of tunning parameter.

Corresponding Author:

Moath Awawdeh
Department of Electrical Engineering, Faculty of Engineering Science, Higher Colleges of Technology
Abu Dhabi, UAE
Email: mawawdeh@hct.ac.ae

1. INTRODUCTION

In this era of emerging global technologies, the global position system (GPS) has advanced reasonably in the recent few years. The system allows the user to obtain beneficial and precise information regarding the positioning and timing notification respective to a user, with the help of appropriate equipment. Since the technology is based on satellite positioning and navigation, it can be accessed globally and universally. The messages and ranging information transmitted by the satellite is obtained by the reception of the GPS signals, which would be helpful in determining the time and position of the location accordingly. From any point on the planet, at any instant of time, at least four satellites can be viewed due to a 24 slot arrangement. In order for this to be possible, in the GPS constellation, the satellites are placed in 6 equally partitioned planes. Every plane would consist of 4 segments, occupied by the satellites. As the satellites are not present in the geosynchronous orbits, the layman would always observe that the geometry of the satellite in the visible sky is constantly changing [1], [2]. Many techniques and methods have been studied and investigated for identifying outliers in positioning determination using GPS [3]-[6]. Based on least square (LS) techniques, many standard receivers have been designed [7]-[9]. All solutions obtained by using the least square method would attempt to reduce the sum of squares of the residual errors between the values obtained from the estimated models and the values obtained from the measurement. The observations read from the equations pertaining to the conventional LS, is presumed to be free of errors, which include both gross and systematic errors [10]-[13]. When the measurements of the global positioning system are affected by the

outliers, the techniques of LS estimation will result in obtaining an estimation that is bad. So the best method to handle outliers is to use a stronger and more robust technique of approach to handle outliers [14]. There are a variety of techniques to choose from but not limited to the following such as generalized M-estimators, the Danish method, R-estimators, MM-estimators, least absolute values method, M-estimators, least median squares, least trimmed squares estimators [3], [4].

Moving-horizon estimation (MHE) technique is performing state estimation by using limited and most recent information. This technique can be widely used for estimating the state and process monitoring. A lot of focus and consideration has been contemplated on MHE due to its rate of success that has been achieved for both linear and non-linear systems in predicting dynamic states and parameters [15]-[18]. The technique of MHE boasts about using a novel method of collecting only a limited extent of data rather than retrieving the entire collection of information from the inception. This technique utilizes only the most recent data to find a solution to the optimization technique, where the state estimation is obtained by minimizing the least-squares function [15], [16], [19], [20]. In the above-mentioned structure, MHE relies on the approach of transforming the problem of estimation using the window of moving fixed size estimation into a program that is quadratic. The quadratic program's size would be determined by the window of fixed size estimation.

GPS signals may contain position errors due to environmental factors or malfunction of subsystems. Environmental factors like reflection, attenuation, scattering of signals may contribute to position errors. When the sensor or system malfunctions, it may contribute to an error in positioning. This paper addresses the position estimation issue of the signal measurements collected from the GPS, which possess outliers. Most of the algorithms that we pursue in solving this problem are used in concurrence to [21], and this method has in turn been derived from the algorithms that use a methodology of comparison [3], [4]. When the pseudo range of a global positioning system signal measurement becomes an outlier, the MHE is used as an estimator to determine the measurements. This paper proposes the model based on the algorithms of [1], [21], [22]. Then the positions are determined or estimated using the proposed technique. The results obtained from this are compared with the results obtained from kalman filtering. The results of the comparison are added to a single value of a synthetic outlier, then the position estimation is performed, and the measurements of pseudo range are obtained. Using the same concept, the outlier identification algorithm and the MHE are used in parallel as in [23], [24], and some other additional measures can be clubbed with the same. The output results obtained from the simulation are exclusively designed for the positioning estimation of the measurements subjected to outliers and free noise of GPS positioning having the original code [21]. In short, there is no new strategy stated here to detect the outliers as such, but the modified moving horizon filter combined with the existing GPS parameters would be conjoined together, the result obtained in the process would be subjected to comparison with the output obtained from the extended Kalman filter (EKF).

The paper is organized as follow: in section 2, we introduce the position estimation with a brief summary of the error model and sources. Section 3, concerns the extended Kalman filtering model which is used for comparison purposes and the proposed moving horizon filter is discussed in section 4. In section 5, simulation example is used with results discussed in section 6. Finally, the conclusion is drawn in section 7.

2. GLOBAL POSITIONING SYSTEM

Position determination is an issue that has to be solved by the observer, which is implemented with the features of receiving and decoding the signals obtained from the GPS. There are 3 dimensions of positioning that have to be solved in free space. When the device starts operating autonomously, it is not always presumed to be in line with perfect synchronizing with the system time of the satellite. In relevance to the concept explained in [1], the non-linear simultaneous equations derived from the system can be used to determine the 4 variables which would represent measurements obtained from 4 different satellites.

$$\rho_1 = \sqrt{(X_1 - x)^2 + (Y_1 - y)^2 + (Z_1 - z)^2} + b \tag{1}$$

$$\rho_2 = \sqrt{(X_2 - x)^2 + (Y_2 - y)^2 + (Z_2 - z)^2} + b \tag{2}$$

$$\rho_3 = \sqrt{(X_3 - x)^2 + (Y_3 - y)^2 + (Z_3 - z)^2} + b \tag{3}$$

$$\rho_4 = \sqrt{(X_4 - x)^2 + (Y_4 - y)^2 + (Z_4 - z)^2} + b \tag{4}$$

Where ρ_i: noiseless pseudo-range, $(X_i\ Y_i\ Z_i)^T$: coordinates of satellite i *(I = 1,...,4),* $(x\ y\ z)^T$: user coordinates, and b: receiver clock. To get the four known parameters, on the (1)-(4) are linearized about an approximate observer location $x_o, = (x_o\ y_0\ z_0)^T$ for *i = 1,..., 4*.

$$\frac{\partial \rho_i}{\partial x} = -\frac{X_i - x_o}{\sqrt{(X_i - x_o)^2 + (Y_i - y_o)^2 + (Z_i - z_o)^2}} \quad (5)$$

$$\frac{\partial \rho_i}{\partial y} = -\frac{Y_i - x_o}{\sqrt{(X_i - x_o)^2 + (Y_i - y_o)^2 + (Z_i - z_o)^2}} \quad (6)$$

$$\frac{\partial \rho_i}{\partial z} = -\frac{Z_i - x_o}{\sqrt{(X_i - x_o)^2 + (Y_i - y_o)^2 + (Z_i - z_o)^2}} \quad (7)$$

The resulting measurement vector equation for pseudo-range as the observable is given in (8) as a noiseless model; where $\hat{\rho}_i(x_{o'})$: predicted pseudo-range, $(\Delta x\ \Delta y\ \Delta z)^T$: difference vector, $x_{o'}$ is the nominal point linearization based on $(x_o\ y_0\ z_0)^T$ and predicted receiver time.

$$\rho_i - \hat{\rho}_i(x_{o'}) = \begin{pmatrix} \frac{\partial \psi_1}{\partial x} & \frac{\partial \psi_1}{\partial y} & \frac{\partial \psi_1}{\partial z} & 1 \\ \frac{\partial \psi_2}{\partial x} & \frac{\partial \psi_2}{\partial y} & \frac{\partial \psi_2}{\partial z} & 1 \\ \frac{\partial \psi_3}{\partial x} & \frac{\partial \psi_3}{\partial y} & \frac{\partial \psi_3}{\partial z} & 1 \\ \frac{\partial \psi_4}{\partial x} & \frac{\partial \psi_4}{\partial y} & \frac{\partial \psi_4}{\partial z} & 1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ b \end{pmatrix} \quad (8)$$

$$\rho_i = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix} \quad (9)$$

$$\hat{\rho}_i(x_{o'}) = \begin{pmatrix} \hat{\rho}_1(x_{o'}) \\ \hat{\rho}_2(x_{o'}) \\ \hat{\rho}_3(x_{o'}) \\ \hat{\rho}_4(x_{o'}) \end{pmatrix} \quad (10)$$

The noiseless pseudo range can be written as

$$\rho_{\mathrm{i}} = \sqrt{(X_{\mathrm{i}} - x)^2 + (Y_{\mathrm{i}} - y)^2 + (Z_{\mathrm{i}} - z)^2} + b \quad (11)$$

2.1. GPS error model and sources

Errors in measurements are classified into noise and bias. Although they both seem the same, noise is a type of error that will change rapidly and returns to a null value within a quick interval. Bias is a type of error that may exist over a period of time. The errors in GPS may occur due to various factors. The navigation message that is broadcasted by the satellite may contain some errors in the values of the parameters can cause errors. When the signal is traveling from the satellite to the receiver, the signal propagation medium varies based on some uncertainties, which affects the signal's traveling time which may contribute to error. Another source of error may be caused due to the receiver's noise [1], [2], [22], [25]-[27]. Some of the significant errors likely to affect a GPS model, like control system error, signal propagation error, and measurement errors, are discussed below.

2.1.1. Control system error

The cause of this type of error is based on the measurements of the satellite clock and the ephemeris. They are calculated at the GPS monitoring station in the control segment. With the utilization of a Kalman filter, the satellite position, satellite velocity, clock bias, frequency bias, and many other such parameters are obtained. Along the three orthogonal directions, the decomposition of the ephemeris error is carried out. Root sum square value of the clock error and line of sight of ephemeris error contributes to the range error of errors in clock and ephemeris [22], [28], [29].

2.1.2. Signal propagation modeling error

These categories of errors are mainly caused due to the changes in carrier phase and code measurement values [1], [22]. Which can briefly: i) signal refraction, dispersive medium, and wave propagation: when the signal of the GPS travels from the satellite to the receiver, the medium of propagation affects the signal properties. Since the signals pass through various layers of the atmosphere, it passes through the ionosphere which consists of charged particles, and it passes through the troposphere which is the neutral portion of the atmosphere. This non-uniformity in the medium of transmission would change the

refractive indices, which may alter the speed of the signal, thus causing errors; and ii) ionospheric and tropospheric delay: The ionosphere tends to be generally stable, but it may tend to exhibit certain fluctuations at areas that are near the poles and the equators. The electron densities tend to vary and change as a result of certain solar storms or flares that can cause errors in signal tracking, as carrier phase and amplitude change. In the troposphere, a delay might be experienced by the signal as a result of the refractive index of air mass. The density of the water vapor changes in accordance with the weather condition, in such conditions the modeling becomes difficult. Also, a dry atmosphere causes a delay [30]-[32].

2.1.3. Measurement error

These types of errors are caused by the power of the signal, the structure of code, and the design of the antenna and receiver. Multipath error, receiver noise error, errors due to measurement models, and user ranges can affect the signal and cause outliers during the process of estimation. Such types of errors are briefed as follow: i) receiver noise and multipath: when a random noise smudges the signal that is responsible for the precision of the code and carrier phase measurement, it is referred to as receiver noise. Many factors may lead to this type of error; some of the factors are antennas or amplifier noise, cable/receiver noise, noise due to signal quantization. If there is no interfering noise present, then the receiver waveform is a sum of the received GPS signal and the fluctuating noise that is random [1], [22], [33]. Multipath causes a change in measurement of phase due to an interfering signal. When the signal approaches the antenna through more than 2 or more paths or ways, the antenna will receive multiple signals, which includes the direct line of sight signal and the various reflected signals. Usually, the multipath error is quasi sinusoidal [33]-[35]; and ii) measurement error model and user range error: the source of the error model is the receiver noise and multipath, which depends on the elevation angle. When the satellite gets to the lower levels of the atmosphere, the power signal reduces, and hence the multipath error tends to increase, which can be controlled with high-end receivers. User range error is the square root of the sum of squares of the range error of control segment, atmospheric propagation, and receiver noise and multipath [30], [36].

3. ESTIMATION STRATEGY AND OUTLIER DETECTION

Estimating the state variables when a system is contaminated by outliers can be treated using various approaches, including MHE and Kalman filtering. As it is well known, Kalman filter is the best estimator in terms of minimizing the quadratic estimation error with white Gaussian disturbances. Extended Kalman filter (EKF), which is the non-linear version of the Kalman filter and the model can be described in (12.a) and (12.b).

$$X_t = f(X_{t-1}) + w_{t-1} \tag{12.a}$$

$$Z_t = h(X_t) + v_t \tag{12.b}$$

Where X_t is the state, Z_t is the measurement, $f(X_{t-1})$ and $h(X_t)$ are the state and measurement functions, respectively, $w_{t-1}$is system noise with covariance Q, and v_t is measurement noise with covariance R. EKF prediction and update are performed in two stages;

– Prediction
  State estimate: $\hat{X}_{t|t-1} = f(\hat{X}_{t-1|t-1})$
  Error covariance: $P_{t|t-1} = F_{t-1}P_{t-1|t-1}F_{t-1}^T + Q_{t-1}$
– Update
  Kalman gain: $K_t = (P_{t|t-1}H_t^T\left(H_tP_{t|t-1}H_t^T\right) + R_t)^{-1}$
  State estimate: $\hat{X}_{t|t} = \hat{X}_{t|t-1} + K_t(z_t - h(\hat{X}_{t|t-1}))$
  Covariance estimate: $P_{t|t} = (I - K_tH_t)P_{t|t-1}$

Extended Kalman filter approximate non-linear function, whereby linearized kalman filter can be used. Figure 1 shows the Kalman filter setup. MHE is a topic that gathered a high degree of contemplation riding on the success model which has been achieved with predictive control. Based on the information proposed by Jazwinski in his pioneering work [37], the basic idea used in MHE for dynamic systems is to perform a state estimation utilizing the limited quantity of most current or recent information. The most important benefit of using the MHE, is its capacity to be accountable for constraints that are present on the state variable [15], [16], [19], [20], [38]-[40]. Outliers prevent the ensuring of a guaranteed performance of an estimator by adding a particular kind of uncertainty in general [41]. In the process of designing filters for any uncertain system, the key requirement to be remembered is the robustness of the system.

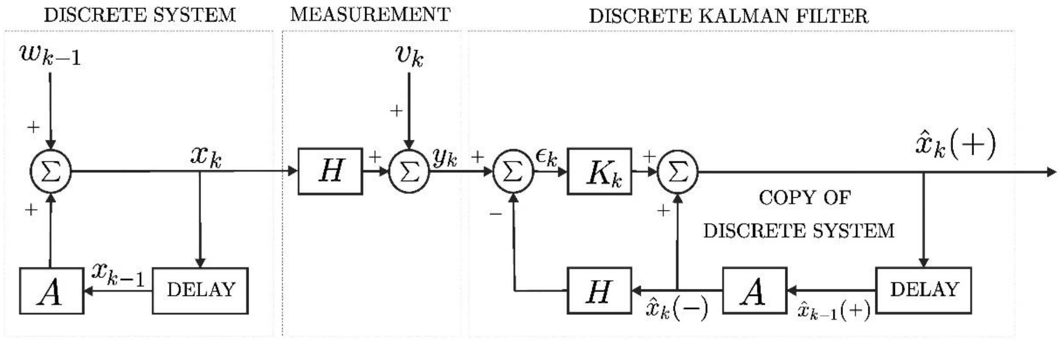


Figure 1. Kalman filter set-up

4. MODIFIED MOVING-HORIZON FILTER FOR POSITIONING ESTIMATION WITH PSEUDO-MEASURE SUBJECT TO OUTLIER

Alessandri *et al*. [23], [24], have developed a novel method for states variable estimation with outlier affecting the measurements equations by minimizing least-squares cost function, where the strategy is to leave out in turn all the measurement that can be affected by the outlier. The design will be used in our simulation considering the non-linear approach, which can be presented for the case in which measurement is corrupted by outlier as follow.

$$J_t^k\left(\hat{x}_{t-N,t}\right) = \mu \left\|\hat{x}_{t-N,t} - \bar{x}_{t-N}\right\|^2 + \frac{1}{N}\sum_{\substack{i=t-N \\ i\neq t-N+k-1}}^{t} \left(y_i - h(\hat{x}_{i,t})\right)^2 \tag{13}$$

For $k = 1,2,\dots,N+1$

where $\mu \geq 0$, the cost in (13) is to be minimized together with the constraints $\hat{x}_{i+1,t} = f\left(\hat{x}_{i,t}, u_i\right)$ for $i = t-N, \dots t-1$. Practically, at each time $t = N, N+1, \dots$ we need to solve $\min_{\hat{x}\ s.t.\ \hat{x}_{i+1,t}=f(\hat{x}_{i,t},u_i)} J_t^k(\hat{x})$ for $k = 0,1,\dots,N+1$ and $\hat{x} \in \mathbb{R}^n$. The estimates of $x_{t-N}, \dots x_t$ at time t is represented as $\hat{x}_{t-N,t}, \hat{x}_{t-N+1,t}, \dots, \hat{x}_{t,t}$. By comparing the optimal cost of both cases (no outlier case and outlier affecting the measurement case), the best cost is associated with estimate, and the approach would be presented as:

$$\begin{aligned} &min_{k=0,1..,N+1}\ min_{\hat{x}\ s.t.\ \hat{x}_{i+1,t}=f(\hat{x}_{i,t},u_i)}\ J_t^k(\hat{x}) \\ &\quad at\ each\ time\ t = N, N+1,\ \dots \end{aligned} \tag{14}$$

Such algorithm considering the output $\hat{x}_{t-N,t}$, can be described as shown in Table 1. The strategy is implemented for simulation purposes following the algorithm in Table 1 and in line with [23], [24], the flowchart of the basic proposed moving-horizon filter (MHF) set up at time t is shown in Figure 2.

Table 1. Outlier MHF algorithm

No.	Description
1	$for\ k = 0\ to\ N+1\ compute\ \hat{x}_{t-N,t}^k$ $\hat{x}_{i+1,t} = f\left(\hat{x}_{i,t}, u_i\right)\ holds\ for\ argmin\ J_t^k(\hat{x}_t)$
2	$end\ for$
3	Choose $k_t^* \in argmin\ J_t^k\left(\hat{x}_{t-N,t}\right)$
4	Update estimate $\hat{x}_{t-N,t} = \hat{x}_{t-N,t}^{k_t^*}$

As shown in Figure 2, the algorithm works on minimizing a set of cost functions (LS), by leaving out all possible outliers within the bath, where at each time instant the lowest cost is determined and hence pushed forward and the process is repeated for all batches (t = N, N+1,…).

5. METHODOLOGY

In this section we address the simulation of position estimation when pseudo-measure is contaminated by outlier. The MHF and EKF proposed in section 3 and 4, respectively, are simulated for the

case where no pseudo-measures are contaminated by outlier, and for the case of outlier occurrence (i.e. single outlier at fixed position). The principle followed here is in line with [1], [2], [21], and considering the previous equations. The system is modeled in (15).

$$\rho_i = \varphi + b + v_i \tag{15}$$

Where $\varphi = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}$, clock bias b, and $\{v_i\}$ is unknown pseudo-range measurement noise which is zero-mean Gaussian except in case of outliers occurrence, which unpredictable and has an unknown and very large covariance. The state, can be defined as in (16).

$$X = (x \quad v_x \quad y \; v_y \quad z \quad v_z \; b \quad d)^T \tag{16}$$

Where position is represented in x, y, z coordinate, $v_x, v_y, and\ v_z$ are the velocity, and d is the clock drift.

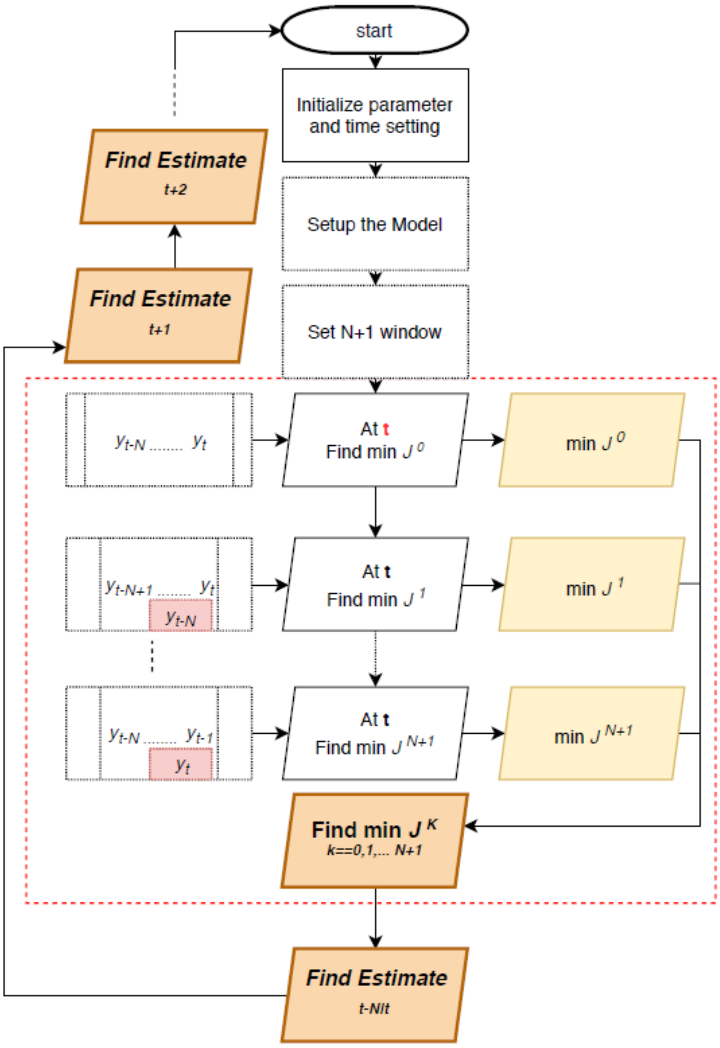


Figure 2. MHF implementation at time t

5.1. Case study

A case of the estimation of position combined with a pseudorange measurement is taken as an outlier, is considered here. The original sampling data are taken from [42], and the noiseless model is from [21]. Let the covariance matrix be defined as:

$$Q = \left(Q_x \ Q_y \ Q_z \ Q_b\right) \tag{17}$$

$$Q_x = \begin{pmatrix} S_{vx} + S_{vx}\frac{T^3}{3} & S_{vx}\frac{T^2}{2} \\ S_{vx}\frac{T^2}{2} & S_{vx}T \end{pmatrix} \quad (18)$$

$$Q_b = \begin{pmatrix} S_f T + S_g\frac{T^3}{3} & S_g\frac{T^2}{2} \\ S_g\frac{T^2}{2} & S_g T \end{pmatrix} \quad (19)$$

Where S_{vx} is the power spectral density of speed noise, S_f is the power spectral density of clock bias noise, and S_f is the power spectral density of frequency drift noise. The initial values of X and $f(.)$ are taken according to [1], [21], and the setting of the measurement noise covariance R is refereed to [43]. The setting of MHF is the same as presented in [23] with additive running criteria. We simulate the pseudo-range and satellite position of a GPS receiver at fixed location for a period of 25 seconds. Synthetic outlier has been added to a pseudo-range measurement at receiving time t = 9 s. For the comparison with EKF, the estimates of the moving-horizon filter obtained by solving $\hat{x}_{t-N,t}$ with the initial setup briefed in Table 2.

Table 2. MHF estimates set-up

Description
$Solve\ \hat{x}_{t-N,t} = \hat{x}_{t-N}^{k(t)}$ $k_t^* \in argmin\, J_t^k(\hat{x}_{t-N,t})\ for\ k = 0,1,..,N+1$
Estimates obtained with $N = 4\ window\ size$ $\mu = 0.6\ Tuning\ parameter$
$For\ 25\ simulation\ run\ (i.e\ T = 25\ steps)$

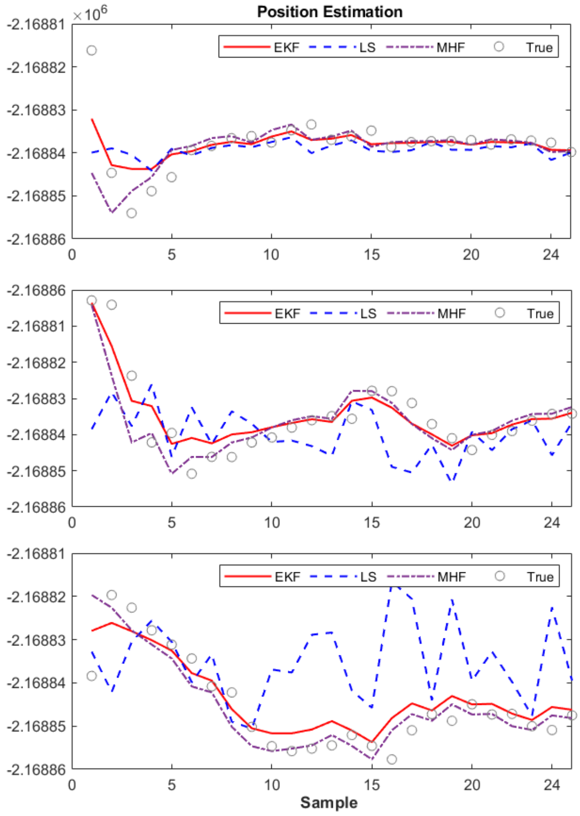
6. RESULTS AND DISCUSSION

The performance of the proposed modified MHF with tuning parameters described in Table 2 is compared with EKF and LS techniques. The positioning estimation where no measurements are affected by outliers (i.e simulating the original sample with zero mean Gaussian noise) is shown in Figure 3(a) for a period of 25 seconds. The positioning error is shown in Figure 3(b) shows that the maximum error recorded for MHF occurred at first 3 readings (due to initial value) with median values 1.45, -0.42, and -2.7 for coordinate (x, y, z) respectively for 25 readings. The case in which the pseudorange measurements are contaminated by outliers is shown in Figure 4 for a case of a synthetic outlier with a random value and fixed position at t = 9 s. The original value is kept in the plotting for comparison reasons, where the arrow indicates the synthetic outlier. The simulation results show that the proposed MHF is more robust to outliers than EKF where the MHF has not been so affected by outlier, still the effect of outliers is observed for the measurements following the time step of the outlier occurrence, which can be improved with more simulation runs and careful choice of the tuning parameter. As shown in Figure 4(a) position estimation [*x*, *y*, *z*] and Figure 4(b) the maximum relative positioning errors were captured for LS and EKF at the step of outlier occurrence and t + 3 following steps. The maximum relative errors for each coordinate are shown in Table 3.

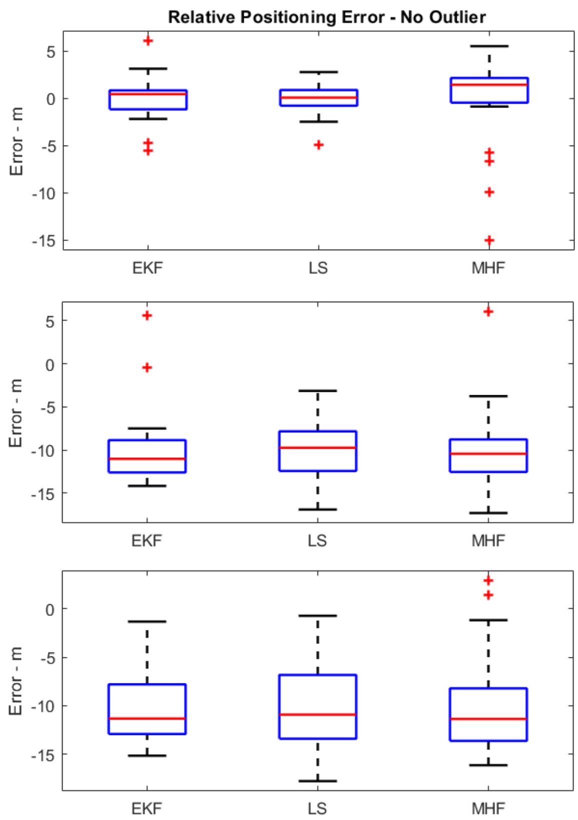
Table 3. Maximum relative error

Step	Coordinate	Filter	Relative error
11	X	LS	93.31
$9_{\|outlier}$	Y	EKF	109.37
10	Z	LS	132.49

Compared with the case of no outliers, the EKF is quite sensitive to outlier value, the highest errors captured at time instant of outlier occurrence, and the following time steps where outlier effect is observed. From the computational time point of view, the total time elapsed is around 60 seconds for 25 simulation runs (for all estimators) with mean computational time (MCT) of EKF around 0.008 s, versus 0.028 s for MHF (the computational time of MHF turns to be demanding comparing to EKF, but the performance turns to be similar over different simulations opposite to EKF which is affected by outlier position, outlier value, and number of runs), such results are varied with window size (N).

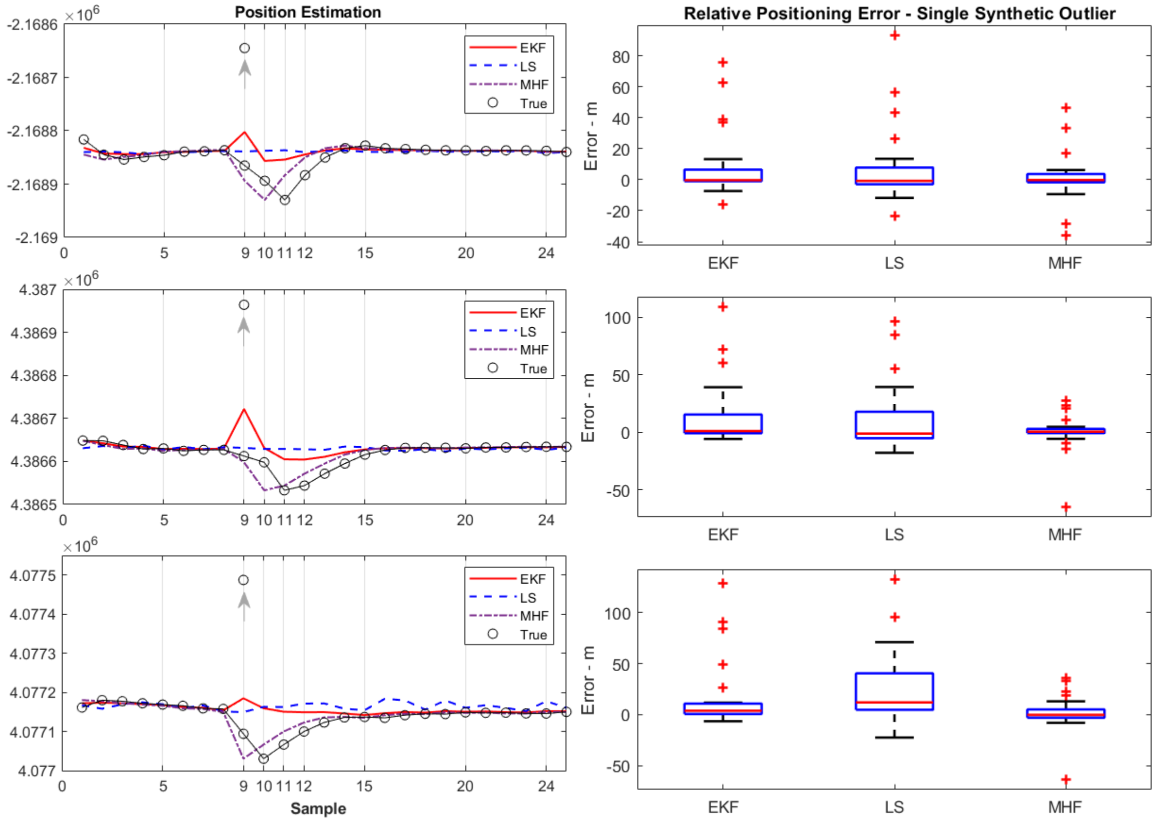


(a)



(b)

Figure 3. The case of no outlier: (a) position estimation [*x*, *y*, *z*] and (b) relative positioning error [*x*, *y*, *z*]



(a) (b)

Figure 4. The case of outlier [t = 9]: (a) position estimation [*x*, *y*, *z*] and (b) relative positioning error [*x*, *y*, *z*]

7. CONCLUSION

This paper proposes applying the concept of moving horizon estimation to the problem of state estimation in which pseudo range measures are affected by outliers. With performed simulation, the results have proved that EKF is turning to be sensitive to outliers opposite to the proposed MHF. The ability of the proposed filter to leave out a measure that is possibly contaminated by outlier is proved by the simulation mean and provided algorithm. The proposed estimator can be easily tuned with the correct selection of tuning parameter, however, the computational time of such estimator is still challenging, which will be studied in future work as well as the effort in reducing the computational time.

ACKNOWLEDGEMENTS

The authors thank the reviewers for their insightful review of our manuscript. This work was supported by the Higher Colleges of Technology (HCT) under Disciplinary Grant No. 113128. United Arab Emirates.

REFERENCES

[1] R. Brown and P. Hwang, *Introduction to random signals and applied Kalman filtering with Matlab exercises*, 4th ed. Hoboken, NJ, USA: John Wiley & Sons, Feb. 2012.
[2] B. H.-Wellenohf, H. Lichtenegger, and J. Collins, *Global positioning system, theory and practice*, 5th ed. Berlin, Heidelberg, D: Springer, 2001, doi: 10.1007/978-3-7091-6199-9.
[3] N. L. Knight and J. Wang, "A comparison of outlier detection procedures and robust estimation methods in GPS positioning," *Journal of Navigation*, vol. 62, no. 4, pp. 699–709, 2009, doi: 10.1017/S0373463309990142.
[4] E. Gökalp, O. Güngör, and Y. Boz, "Evaluation of different outlier detection methods for GPS networks," *Sensor*, vol. 8, no. 11, pp. 7344–7358, Nov. 2008, doi: 10.3390/s8117344.
[5] C. Yang, W. Shi, and W. Chen, "Robust M–M unscented Kalman filtering for GPS/IMU navigation," *Journal of Geodesy*, vol. 93, no. 8, pp. 1093–1104, Jan 2019, doi: 10.1007/s00190-018-01227-5.
[6] Y. Zhang, F. Wu, and H. Isshiki, "A new cascade method for detecting GPS multiple outliers based on total residuals of observation equations," *Proceedings of the 2012 IEEE/ION Position, Location and Navigation Symposium*, 2012, pp. 208-215, doi: 10.1109/PLANS.2012.6236882.

[7] M. A. Akram, P. Liu, Y. Wang, and J. Qian, "GNSS positioning accuracy enhancement based on robust statistical MM estimation theory for ground vehicles in challenging environments," *Journal of Applied Sciences*, vol. 8, no. 6, p. 876, 2018, doi: 10.3390/app8060876.

[8] E. Kaplan and C. Hegarty, *Understanding GPS/GNSS: Principles and Applications*, Third Edition, Artech, 2017.

[9] A. Leick, L. Rapoport, and D. Tatarnikov, *GPS Satellite Surveying*, 4th ed. Hoboken, NJ, USA:Wiley, Mar. 2015.

[10] K. Fallahi, C. Cheng and M. Fattouche, "Robust Positioning Systems in the Presence of Outliers Under Weak GPS Signal Conditions," in *IEEE Systems Journal*, vol. 6, no. 3, pp. 401-413, Sep. 2012, doi: 10.1109/JSYST.2011.2173622.

[11] N. T. Duong, "Applying the least square method to calculate the non-simultaneous errors of ship's position determined by global positioning system (GPS)," *Journal of Physics: Conference Series*, vol. 1679, no. 5, p. 052015, 2020, doi: 10.1088/1742-6596/1679/5/052015.

[12] R. Revathi, K. S. Ramesh, S. K. Rao, and K. U. Kiran, "Application of least squares algorithm for precise GPS receiver positioning," in *Information and Decision Sciences. Advances in Intelligent Systems and Computing*, S. C. Satapathy, eds. Beach Rd, Gateway East, SG: Springer, vol 701, 2018, doi: 10.1007/978-981-10-7563-6_31.

[13] D. Jwo, M. Hsieh, and Y. Lee, "GPS navigation solution using the iterative least absolute deviation approach," *Scientia Iranica*, vol. 22, no. 6, pp. 2103-2111, Dec. 2015. [Online]. Available: http://scientiairanica.sharif.edu/article_3758_ad3fc490c83c605ac9bd687334932cd7.pdf

[14] X.-W. Chang and Y. Guo, "Huber's M-estimation in relative GPS positioning: computational aspects," *Journal of Geodesy*, vol. 79, no. 6-7 pp. 351–362, Aug. 2005, doi: 10.1007/s00190-005-0473-y.

[15] J. B. Rawlings, "Moving Horizon Estimation," in *Encyclopedia of Systems and Control*, J. Baillieul and T. Samad Eds. Berlin, Heidelberg, D: Springer, 2013, doi: 10.1007/978-1-4471-5102-9_4-1.

[16] A. Alessandri, M. Baglietto, and G. Battistelli, "Receding-horizon estimation for discrete-time linear systems," in *IEEE Transactions on Automatic Control*, vol. 48, no. 3, pp. 473-478, Mar. 2003, doi: 10.1109/TAC.2003.809155.

[17] C. V. Rao and J. B. Rawlings, "Constrained process monitoring: A moving horizon approach," *AIChE Journal*, vol. 48, no. 1, pp. 97–109, 2002, doi: 10.1002/aic.690480111.

[18] B. Boulkroune, M. Darouach, and M. Zasadzinski, "Moving horizon state estimation for linear discrete-time singular systems," *IET Control Theory and Applications*, vol. 4, no. 3, pp. 339–350, 2010, doi: 10.1049/iet-cta.2008.0280.

[19] J. Brembeck, "Nonlinear constrained moving horizon estimation applied to vehicle position estimation," *Sensors*, vol. 19, no. 10, p. 2276, May 2019, doi: 10.3390/s19102276.

[20] A. Alessandri, M. Baglietto, and G. Battistelli, "Moving-horizon state estimation for non-linear discrete-time systems: New stability results and approximation schemes," *Automatica*, vol. 44, no. 7, pp. 1753–1765, 2008, doi: 10.1016/j.automatica.2007.11.020.

[21] Y. Wang, "Position estimation using extended Kalman Filter and RTS-smoother in a GPS receiver," *2012 5th International Congress on Image and Signal Processing*, 2012, pp. 1718-1721, doi: 10.1109/CISP.2012.6469979.

[22] P. Misra and P. Enge, *Global positioning system signals, measurements, and performance*, Lincoln, MA, USA: Ganga-Jamuna Press, 2010.

[23] A. Alessandri and M. Awawdeh, "Moving-horizon estimation for discrete-time linear systems with measurements subject to outliers," *53rd IEEE Conference on Decision and Control*, 2014, pp. 2591-2596, doi: 10.1109/CDC.2014.7039785.

[24] A. Alessandri and M. Awawdeh "Moving-horizon estimation with guaranteed robustness for discrete-time linear systems and measurements subject to outliers," *Automatica*, vol. 67, pp. 85–93, 2016, doi: 10.1016/j.automatica.2016.01.015.

[25] D. Cooksey, "Understanding the global positioning system (GPS)," *Montana State University-Bozeman*, 2014. [Online]. Available: https://www.scribd.com/document/238442280/Understanding-the-Global-Positioning-System.

[26] A. El Abbous and N. Samanta, "A modeling of GPS error distributions," *2017 European Navigation Conference (ENC)*, 2017, pp. 119-127, doi: 10.1109/EURONAV.2017.7954200.

[27] S. Liu, V. Elangovan and W. Xiang, "A Vehicular GPS Error Prediction Model Based on Data Smoothing Preprocessed LSTM," *2019 IEEE 90th Vehicular Technology Conference (VTC2019-Fall)*, 2019, pp. 1-5, doi: 10.1109/VTCFall.2019.8891454.

[28] D. Sathyamoorthy, S. Shafii, Z. F. M. Amin, A. Jusoh, and S. Z. Ali, "Evaluating the effect of global positioning system (GPS) satellite clock error via GPS simulation," in *8th IGRSM International Conference and Exhibition on Geospatial & Remote Sensing (IGRSM 2016)*, Apr. 2016, vol. 37, doi: 10.1088/1755-1315/37/1/012013.

[29] N. M. Vani and S. Banu, "Evaluation of GPS Data for Navigational Solution and Error Reduction Using Kalman Filter," 2018 International Conference on Design Innovations for 3Cs Compute Communicate Control (ICDI3C), 2018, pp. 12-17, doi: 10.1109/ICDI3C.2018.00012.

[30] D. Tang, D. Lu, B. Cai, and J. Wang, "GNSS Localization Propagation Error Estimation Considering Environmental Conditions," *2018 16th International Conference on Intelligent Transportation Systems Telecommunications (ITST)*, 2018, pp. 1-7, doi: 10.1109/ITST.2018.8566771.

[31] J. Li and Y. Wang, "GPS receiver failure detection method in high dynamic environment," *2018 IEEE/ION Position, Location and Navigation Symposium (PLANS)*, 2018, pp. 349-354, doi: 10.1109/PLANS.2018.8373400.

[32] T. Hayakawa, N. Murai, and T. Sugiyama, "GPS Calculated Distance Error Improvement by Planar Approximation of Pseudo Range in Many Skyscrapers Environments," *2019 IEEE 90th Vehicular Technology Conference (VTC2019-Fall)*, 2019, pp. 1-5, doi: 10.1109/VTCFall.2019.8891219.

[33] J-M. Zogg, *GPS: Essentials of satelite navigation*, 2009. [Online]. Available: https://www.u-blox.com/sites/default/files/products/documents/GPS-Compendium_Book_%28GPS-X-02007%29.pdf

[34] Y. Kamatham, "Estimation, analysis and prediction of multipath error for static GNSS applications," *2018 Conference on Signal Processing And Communication Engineering Systems (SPACES)*, 2018, pp. 62-65, doi: 10.1109/SPACES.2018.8316317.

[35] A. Pirsiavash, A. Broumandan, G. Lachapelle, and K. O'Keefe, "Detection and De-weighting of Multipath-affected Measurements in a GPS/Galileo Combined Solution," *2019 European Navigation Conference (ENC)*, 2019, pp. 1-11, doi: 10.1109/EURONAV.2019.8714191.

[36] A. A. M. S. Sadman and M. Hossam-E-Haider, "GNSS Position Accuracy Considering GDOP and UERE for Different Constellation over Bangladesh," *2019 22nd International Conference on Computer and Information Technology (ICCIT)*, 2019, pp. 1-5, doi: 10.1109/ICCIT48885.2019.9038577.

[37] A. Jazwinski, "Limited memory optimal filtering," in *IEEE Transactions on Automatic Control*, vol. 13, no. 5, pp. 558-563, October 1968, doi: 10.1109/TAC.1968.1098981.

[38] G. Ferrari-Trecate, D. Mignone, and M. Morari, "Moving horizon estimation for hybrid systems," in *IEEE Transactions on Automatic Control*, vol. 47, no. 10, pp. 1663-1676, Oct. 2002, doi: 10.1109/TAC.2002.802772.

[39] C. V. Rao, J. B. Rawlings, and D. Q. Mayne, "Constrained state estimation for nonlinear discrete-time systems: stability and moving horizon approximations," in *IEEE Transactions on Automatic Control*, vol. 48, no. 2, pp. 246-258, Feb. 2003, doi: 10.1109/TAC.2002.808470.

[40] A. Alessandri, M. Baglietto, G. Battistelli and M. Gaggero, “Moving-Horizon State Estimation for Nonlinear Systems Using Neural Networks,” in *IEEE Transactions on Neural Networks*, vol. 22, no. 5, pp. 768-780, May 2011, doi: 10.1109/TNN.2011.2116803.
[41] A. Matasov and V. Samokhvalov, “Guaranteeing parameter estimation with anomalous measurement errors,” *Automatica*, vol. 32, no. 9, pp. 1317–1322, Sep. 1996, doi: 10.1016/0005-1098(96)00064-7.
[42] K. Borre, D. M. Akos, N. Betersen, P. Rinder, and S. H. Jensen, *A software-Defined GPS and Galileo Receiver*, Basel, CH: Birkhäuser, Aug. 2006. [Online]. Available: https://www.ocf.berkeley.edu/~marsy/resources/gnss/A%20Software-Defined%20GPS%20and%20Galileo%20Receiver.pdf
[43] R. Leonardo, and M. Santos, “An Empirical Stochastic Model for GPS,” *Dynamic Planet*, vol. 130, pp. 179-185, Feb. 2007, doi: 10.1007/978-3-540-49350-1_28.

BIOGRAPHIES OF AUTHORS

Moath Awawdeh IEEE (CSS), received the B.S. in communication and software engineering from Al Balqa Applied University for Engineering Technology, Jordan, in 2009, and Ph.D. in mathematical engineering and simulation with specialization in control engineering from the University of Genoa, Italy, in 2014. His main research interests include outlier detection, data mining, machine learning, signal processing, and control engineering applications. He is currently working on the problem of outlier detection and correction in LTI systems with a specific approach of state estimation and measurements validation. He can be contacted at email: mawawdeh@hct.ac.ae.

Tarig Faisal Ibrahim Dr. Tarig Faisal received his master degree of Mechatronics Engineering from IIUM university in 2006 and his Ph.D degree in Signal Processing from University of Malaya, Malaysia, in 2011. He has been the Dean of Academic Operations at Higher Colleges of Technology since 2018. His research focuses on biomedical signal processing, intelligent systems, robotics, control, embedded system design, IoTs, machine learning, and outcome based education. He has been a reviewer for multiple journals including IEEE, Elsevier, Taylor and Francis, and Springer Nature. He has more than 20 years of academic and industry experience of which he worked as an engineering, assistant professor, Programs Chair, Head of department, and Division Chair. He is also a charted engineering as well as Senior Fellow of Higher Education Academy. He can be contacted at email: tfaisal@hct.ac.ae.

Anees Bashir received Bachelor in electronics and communication engineering from Anna University, India in 2012, and master degree in applied electronics engineering from M.S. University India, in 2014. Her main research interests include embedded system design, signal processing, outlier detection and data mining. She is currently working on her research based on Moving-horizon strategy for anomaly detection in discrete LTI systems. She can be contacted at email: abashir1@hct.ac.ae.

Flower M. Queen Associated professor, has completed Ph. D from the Anna university and has completed Masters in Applied Electronics from the RVS college of engineering and technology. She has 20+ years of experience in academia and research with interest of control systems. She is currently involved in supervising 4 Ph. D. students at her department at NIC higher education. She can be contacted at email: mpflowerqueen@gmail.com.